Possibilistic information fusion by maximal coherent subsets

S. Destercke ¹ D. Dubois ² and E. Chojnacki ¹

¹Institute of radioprotection and nuclear safety
Cadarache, France

²Toulouse institute of computer science
University Paul-Sabatier

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Motivations

There are many areas or cases in which one has to merge the information delivered by multiple sources, in order to analyze or synthesize it:

- Expert opinions (risk, reliability or financial analysis, . . . )
- Multi-sensors (robotics, military detection, industrial processes, . . . )
- Data base fusion
- Image processing (medical application, . . . )
Uncertainty theories allow to model incomplete information and to aggregate it with a wide catalog of operators.

Three basic fusion operators

- Conjunction (Intersection): make the assumption that all sources are reliable.
- Disjunction (Union): make the assumption that at least one source is reliable.
- Arithmetic mean (Compromise): Similar to statistical counting, assuming that each source is an independent observation of the same quantity.
Unfortunately, when sources are more than two, none of this basic operators is generally satisfactory!

- Conflict is often present so conjunction results in the empty set.
- Disjunction give very reliable results, unfortunately often too imprecise to be really useful.
- Hypothesis made when using arithmetic mean (independence of sources) are seldom satisfied.

Need to cope with the conflict in an adaptative and efficient manner.
Dealing with conflict: how?

Basic idea

Adapt methods used in logic to handle inconsistency to numerical and quantitative framework (here, possibility distributions).

Maximal Coherent Subsets method

When a set of logic formulas is inconsistent, a way to handle the inconsistency is to:

- Consider all maximal subsets of formulas that are consistent (i.e. take the conjunctions of consistent formula).
- Consider a proposition as true if it is true in all the consistent subsets (i.e. a proposition is derivable from an inconsistent base if it is true in all models of all its consistent subsets).
MCS on intervals: illustration

Result of MCS method: \((I_1 \cap I_2) \cup (I_2 \cap I_3 \cap I_4)\)

Computational complexity

- In logic: finding Max. Co. Sub. is exponential in complexity
- In numerical framework: finding Max. Co. Sub. is linear in complexity (thanks to the natural ordering of numbers)
If we have $n$ distributions $\pi_i$, the set of $\alpha$-cuts for a given $\alpha$ is a set of $n$ intervals $\Rightarrow$ we can apply MCS method on them.
For two different levels $\alpha < \beta$, the resulting sets $E_\alpha, E_\beta$ may fail to be nested ($E_\alpha \subseteq E_\beta$), since we can go from conjunction to disjunction of intervals.

But there will be a finite number of levels $\beta_i \ i = 0, 1, \ldots, n$ s.t. resulting $E_\alpha$ will be nested for $\alpha \in (\beta_i, \beta_{i+1}]$.
MCS on possibility distributions: how? (2)

For two different levels $\alpha < \beta$, the resulting sets $E_\alpha, E_\beta$ are not forcefully nested ($E_\alpha \subseteq E_\beta$), since we can go from conjunction to disjunction of intervals.

But there will be a finite number of levels $\beta_i$ $i = 0, 1, \ldots, n$ s.t. resulting $E_\alpha$ will be nested for $\alpha \in (\beta_i, \beta_{i+1}]$. 

\[
\begin{align*}
\beta_0 &= 0 \\
\beta_1 \\
\beta_2 \\
\beta_3 &= 11
\end{align*}
\]
MCS on possibility distributions: the result.

The result can be seen as a fuzzy belief structure (formally equivalent to a fuzzy random variable) where each (normalized) fuzzy set $F_i$ has mass $m(F_i) = \beta_i - \beta_{i-1}$.
Information is summarized by a mathematically founded method, with a good balance between information gain and reliability.

Since their introduction by Zadeh in 1979, fuzzy belief structures*, have been studied by various authors:

- J. Yen, T. Denoeux, R. Yager as extensions of usual belief functions
- C. Baudrit, I. Couso, D. Dubois (IJAR 2007) as a model of imprecise probabilities

The results of the SMC approach can be analyzed in the light of the above settings

* Fuzzy belief structures are special kinds of fuzzy random variables introduced by Feron (1976), Kwakernaak (1978), later studied by Puri and Ralescu (1987), Kruse (1987), etc.
Once the fuzzy belief structure is computed, we propose indices based on these results to measure:

- The gain in precision (with fuzzy cardinalities, gradual numbers)
- The confidence in an event, a source (with generalized belief/plausibility functions and pignistic probability)
- The confusion between sources (with Shannon/Hartley like measures)

To compute a final synthetic possibility distribution, we propose to take the contour function (i.e. plausibility of single values). This comes down to computing the weighted arithmetic mean of fuzzy sets $F_i$ with weights $m(F_i)$. 
Post-treatment: computing the contour function
Conclusions

We have presented a fusion method that

- use a well founded logical approach to deal with conflict
- takes all sources into account and does not need extra information
- is computationally simple (uses only linear algorithms)
- can use the formal setting of fuzzy belief structures and fuzzy random variables to analyze the information (and the sources)

Perspectives

- Integrating available additional information to the method (i.e. take source reliability and the metric into account)
- Comparison (axiomatic, practical) with other fusion rules (Oussalah et al., Delmotte)
- Practical application to nuclear computer codes (BEMUSE project)