Possibilistic information fusion by maximal coherent subsets

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Motivations

There are many areas or cases in which one has to merge the information delivered by multiple sources, in order to analyze or synthesize it:

- Expert opinions (risk, reliability or financial analysis, . . .)
- Multi-sensors (robotics, military detection, industrial processes, . . .)
- Data base fusion
- Image processing (medical application, . . .)

Basic tools

Uncertainty theories allow to model incomplete information and to aggregate it with a wide catalog of operators.

Three basic fusion operators

- Conjunction (Intersection): make the assumption that **all** sources are reliable.
- Disjunction (Union): make the assumption that **at least** one source is reliable
- Arithmetic mean (Compromise): Similar to statistical counting, assuming that each source is an independent observation of the same quantity.

Dealing with conflict : why ?

Unfortunately, when sources are more than two, none of this basic operators is generally satisfactory!

- Conflict is often present so conjunction results in the empty set.
- Disjunction give very reliable results, unfortunately often too imprecise to be really useful
- Hypothesis made when using arithmetic mean (independence of sources) are seldom satisfied.

► Need to cope with the conflict in an adaptative and efficient manner.

Dealing with conflict : how ?

Basic idea

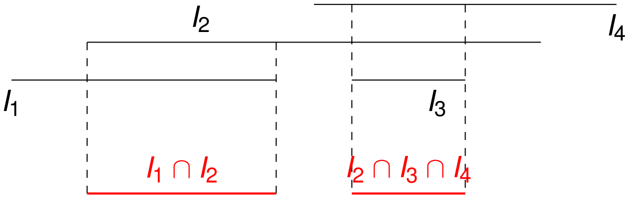
Adapt methods used in logic to handle inconsistency to numerical and quantitative framework (here, possibility distributions).

Maximal Coherent Subsets method

When a set of logic formulas is inconsistent, a way to handle the inconsistency is to:

- Consider all maximal subsets of formulas that are consistent (i.e. take the conjunctions of consistent formula).
- Consider a proposition as true if it is true in all the consistent subsets (i.e. a proposition is derivable from an inconsistent base if it is true in all models of all its consistent subsets).

MCS on intervals : illustration



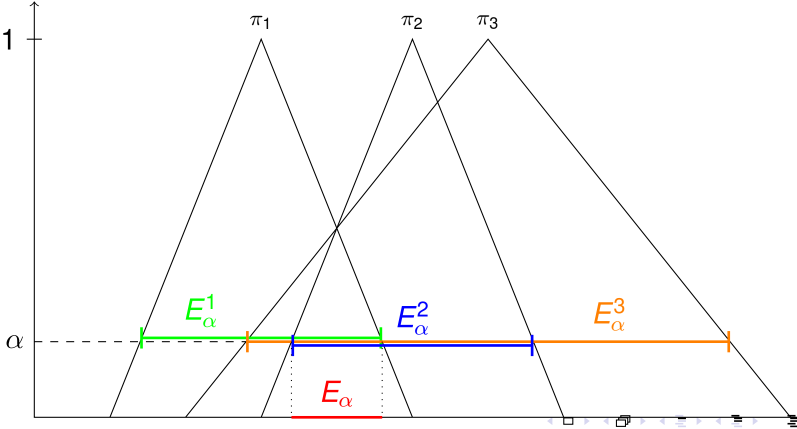
Result of MCS method: $(I_1 \cap I_2) \cup (I_2 \cap I_3 \cap I_4)$

Computational complexity

- In logic : finding Max. Co. Sub. is exponential in complexity
- In numerical framework : finding Max. Co. Sub. is linear in complexity (thanks to the natural ordering of numbers)

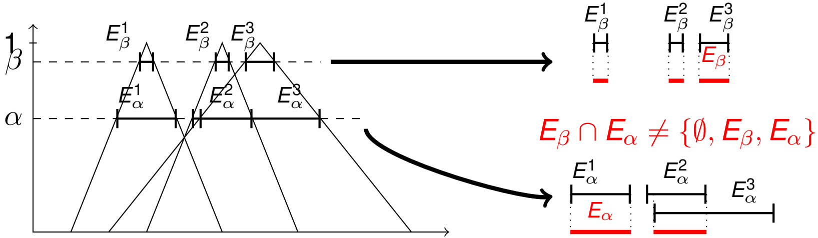
MCS on possibility distributions : how ? (1)

If we have n distributions π_i, the set of α-cuts for a given α is a set of n intervals $\Rightarrow$ we can apply MCS method on them.



MCS on possibility distributions : how ? (2)

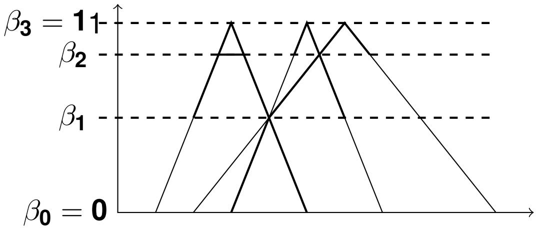
For two different levels $\alpha < \beta$, the resulting sets E_α, E_β may fail to be nested ($E_\alpha \subseteq E_\beta$), since we can go from conjunction to disjunction of intervals



But there will be a finite number of levels β_i $i = 0, 1, \ldots, n$ s.t. resulting E_α will be nested for $\alpha \in (\beta_i, \beta_{i+1}]$

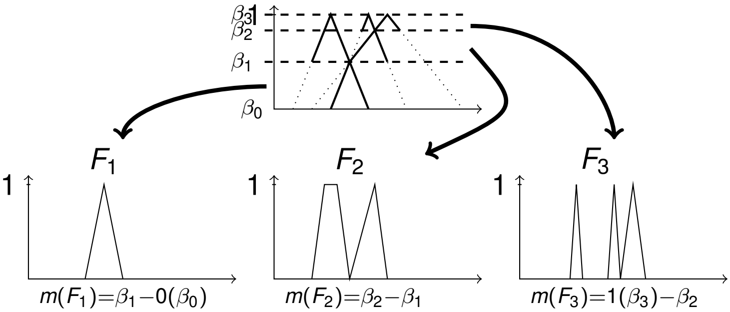
MCS on possibility distributions : how ? (2)

For two different levels $\alpha < \beta$, the resulting sets E_α, E_β are not forcefully nested ($E_\alpha \subseteq E_\beta$), since we can go from conjunction to disjunction of intervals



But there will be a finite number of levels β_i $i = 0, 1, \ldots, n$ s.t. resulting E_α will be nested for $\alpha \in (\beta_i, \beta_{i+1}]$

MCS on possibility distributions : the result.



The result can be seen as a fuzzy belief structure (formally equivalent to a fuzzy random variable) where each (normalized) fuzzy set F_i has mass $m(F_i) = \beta_i - \beta_{i-1}$.

Existing results

- Information is summarized by a mathematically founded method, with a good balance between information gain and reliability.
- Since their introduction by Zadeh in 1979, fuzzy belief structures*, have been studied by various authors:
  - J.Yen, T.Denoeux, R. Yager as extensions of usual belief functions
  - C. Baudrit, I. Couso, D. Dubois (IJAR 2007) as a model of imprecise probabilities

The results of the SMC approach can be analyzed in the light of the above settings

* Fuzzy belief structures are special kinds of fuzzy random variables introduced by Feron (1976), Kwakernaak (1978), later studied by Puri and Ralescu (1987), Kruse (1987), etc.

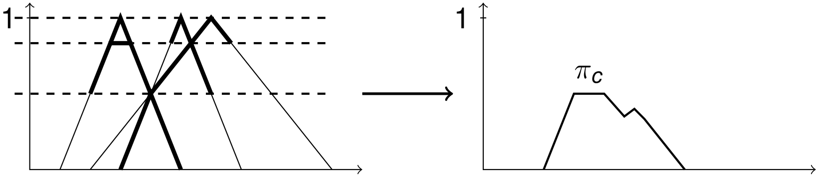
Post-treatment of summarized information

Once the fuzzy belief structure is computed, we propose indices based on these results to measure:

- The gain in precision (with fuzzy cardinalities, gradual numbers)
- The confidence in an event, a source (with generalized belief/plausibility functions and pignistic probability)
- The confusion between sources (with Shannon/Hartley like measures)

To compute a final synthetic possibility distribution, we propose to take the contour function (i.e. plausibility of single values). This comes down to computing the weighted arithmetic mean of fuzzy sets F_i with weights $m(F_i)$.

Post-treatment : computing the contour function



Conclusions and perspective

Conclusions

We have presented a fusion method that

- use a well founded logical approach to deal with conflict
- takes all sources into account and does not need extra information
- is computationally simple (uses only linear algorithms)
- can use the formal setting of fuzzy belief structures and fuzzy random variables to analyze the information (and the sources)

Perspectives

- Integrating available additional information to the method (i.e. take source reliability and the metric into account)
- Comparison (axiomatic, practical) with other fusion rules (Oussalah et al., Delmotte)
- Practical application to nuclear computer codes (BEMUSE project)